AD-A123 916 WAVE PROPAGATION-NONLINEAR BOUNDARY VALUE PROBLEMS(U) 1/1
MIAMI UNIV CORAL GABLES FLA DEPT OF MATHEMATICS
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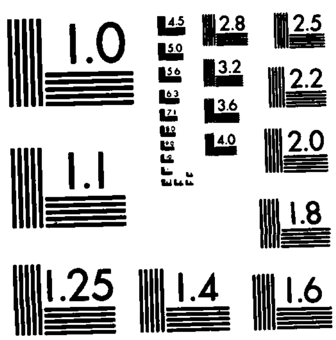

MICROCOPY RESOLUTION TEST CHART
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Annual Report

For

Navy Contract N00014-78-C-0608

(UM A/C BR6537)

"Wave Propagation-Nonlinear Boundary Value Problems"

Principal Investigatior: R.A. Goldstein

25 January 1982

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Work ~~was completed~~ on a paper entitled "Perturbed Bifurcation of Stationary Striations in a Contaminatied, Non-Uniform Plasma." It was accepted by SIAM APPLIED MATH and is currently in press. The ~~paper~~ is attached. included

On going research in collaboration with J. Magnan (Northwestern) continues with respect to moving stuations or moduluted travelling waves in reaction diffusion systems.

Goldstein and Magnan

Moving Striations

Problem

$$\frac{\partial N}{\partial t} = D_- \frac{\partial^2 N}{\partial z^2} + \frac{\mu_+}{\mu_-}\frac{J}{e}\left(\frac{P}{N^2}\frac{\partial N}{\partial z} - \frac{1}{N}\frac{\partial P}{\partial z}\right) + F ,$$

$$\frac{\partial P}{\partial t} = D_+ \frac{\partial^2 P}{\partial z^2} + \frac{\mu_+}{\mu_-}\frac{J}{e}\left(\frac{P}{N^2}\frac{\partial N}{\partial z} - \frac{1}{N}\frac{\partial P}{\partial z}\right) + G ,$$

$$\frac{\partial M}{\partial t} = D^* \frac{\partial^2 M}{\partial z^2} + H ,$$

where it has been assumed that $\mu_- N \gg \mu_+ P$

B.C. (see Fig 1)

$$N(0) - \alpha \frac{\partial N(0)}{\partial z} = A \quad , \quad N(L) + \alpha \frac{\partial N(L)}{\partial z} = A$$

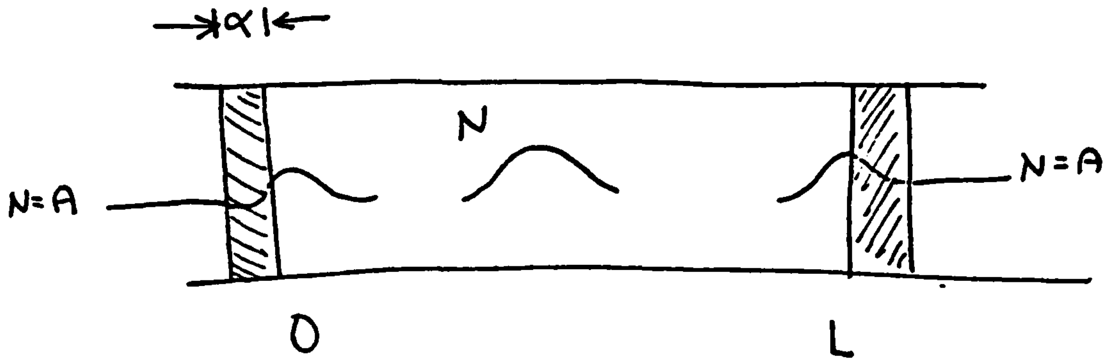
$$P(0) - \beta \frac{\partial P(0)}{\partial z} = B \quad , \quad P(L) + \beta \frac{\partial P(L)}{\partial z} = B$$

$$M(0) - \gamma \frac{\partial M(0)}{\partial z} = C \quad , \quad M(L) + \gamma \frac{\partial M(L)}{\partial z} = C$$

I.C.

$$N(z,0) = N_0$$
$$P(z,0) = P_0$$
$$M(z,0) = M_0$$

Fig 1



Scaled equations

$$\frac{\partial n}{\partial \tau} = \epsilon \frac{\partial^2 n}{\partial \xi^2} + \frac{p}{n^2}\frac{\partial n}{\partial \xi} - \frac{1}{n}\frac{\partial p}{\partial \xi} + f$$

$$\frac{\partial p}{\partial \tau} = \epsilon \delta \frac{\partial^2 p}{\partial \xi^2} + \frac{p}{n^2}\frac{\partial n}{\partial \xi} - \frac{1}{n}\frac{\partial p}{\partial \xi} + g$$

$$\frac{\partial m}{\partial \tau} = \epsilon \theta \frac{\partial^2 m}{\partial \xi^2} + h$$

B.C.

$$n(0,\tau) - \epsilon \frac{\partial n(0,\tau)}{\partial \xi} = a = n(\ell,\tau) + \epsilon \frac{\partial n(\ell,\tau)}{\partial \xi}$$

$$p(0,\tau) - \epsilon \frac{\partial p(0,\tau)}{\partial \xi} = b = p(\ell,\tau) + \epsilon \frac{\partial p(\ell,\tau)}{\partial \xi}$$

$$m(0,\tau) - \epsilon \frac{\partial m(0,\tau)}{\partial \xi} = c = m(\ell,\tau) + \epsilon \frac{\partial m(\ell,\tau)}{\partial \xi}$$

I.C.

$n(\xi,0)=1$

$p(\xi,0)=1$

$m(\xi,0)=1$

where

$$z = \left(\frac{L^2}{D_-}\right) \frac{\mu_+}{\mu_-} \frac{J}{eN_0} \xi \quad ,$$

$$t = \frac{L^2}{D_-} \tau \quad ,$$

$$\epsilon = \left(\frac{\frac{D_- N_0}{L}}{\frac{\mu_+}{\mu_-} \frac{J}{e}} \right)^2 = \left(\frac{\Gamma_D}{\Gamma_+} \right)^2 \quad ,$$

$$N = nN_0 \ , \ P = p\dot{N}_0 \ , \ M = mM_0 \ ,$$
$$A = aN_0 \ , \ B = bN_0 \ , \ C = cM_0 \ ,$$

$$\alpha = \beta = \gamma = \sqrt{\epsilon}\, L \ ,$$

$$\delta = \left(\frac{D_+}{D_-}\right)^2 \ , \ \theta = \left(\frac{D^*}{D_-}\right)^2 \ , \ \ell = \sqrt{\epsilon} \ .$$

Xform to homogeneous B.C.

let $n = n' + a$

$p = p' + b$

$m = m' + c$

substitute into the scaled equations and drop the primes.

$$\frac{\partial n}{\partial \tau} = \epsilon \frac{\partial^2 n}{\partial \xi^2} + \frac{(p+b)}{(n+a)^2}\frac{\partial n}{\partial \xi} - \frac{1}{(n+a)}\frac{\partial p}{\partial \xi} + f$$

$$\frac{\partial p}{\partial \tau} = \epsilon\delta \frac{\partial^2 p}{\partial \xi^2} + \frac{(p+b)}{(n+a)^2}\frac{\partial n}{\partial \xi} - \frac{1}{(n+a)}\frac{\partial p}{\partial \xi} + g$$

$$\frac{\partial m}{\partial \tau} = \epsilon\theta \frac{\partial^2 m}{\partial \xi^2} + h$$

BC:

$$n(0,\tau) - \epsilon\frac{\partial n(0,\tau)}{\partial \xi} = 0 = n(\ell,\tau) + \epsilon\frac{\partial n(\ell,\tau)}{\partial \xi}$$

$$p(0,\tau) - \epsilon\frac{\partial p(0,\tau)}{\partial \xi} = 0 = p(\ell,\tau) + \epsilon\frac{\partial p(\ell,\tau)}{\partial \xi}$$

$$m(0,\tau) - \epsilon\frac{\partial m(0,\tau)}{\partial \xi} = 0 = m(\ell,\tau) + \epsilon\frac{\partial m(\ell,\tau)}{\partial \xi}$$

IC:

$n(\xi,0) = 1 - a$

$p(\xi,0) = 1 - b$

$m(\xi,0) = 1 - c$

Try a solution

let
$$n(\xi,\tau) = e^{-\frac{\phi(\xi,\tau)}{2\epsilon}} x(\xi,\tau,\epsilon),$$
$$P(\xi,\tau) = e^{-\frac{\phi(\xi,\tau)}{2\epsilon}} y(\xi,\tau,\epsilon),$$
$$m(\xi,\tau) = e^{-\frac{\phi(\xi,\tau)}{2\epsilon}} z(\xi,\tau,\epsilon),$$
substitute into equations and get to order $\frac{1}{\epsilon}$:

$$\phi_\tau + \tfrac{1}{2}\phi_\xi^2 = 0$$

with solution

$$\phi(\xi,\tau) = \frac{1}{2}\frac{(\xi-\xi_0)^2}{\tau}.$$

$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ will satisfy the 1st order unperturbed problem (hence a travelling wave)

We will seek "modulated" travelling wave solutions